

# The evolution of the Mizar Mathematical Library

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W i t a m y

# The Mizar Project

- Mizar
- MML – the repository of computer checked mathematics
- The structure of the MML
- Writing Mizar articles
- Maintaining MML

# Selected theorems

- Abian's fixed point theorem
- Birkhoff's theorem for many-sorted algebras
- König's theorem
- Deduction theorem
- Dynkin's lemma
- Euler's theorem and "Small" Fermat's theorem

- Properties of Fibonacci numbers
- Minimization of finite-state machines
- Dijkstra's Shortest Path Algorithm
- Hahn-Banach theorem
- Lattice of substitutions as a Heyting algebra
- Hilbert basis theorem
- Fundamental theorem of algebra
- Robbins algebras vs. boolean algebras

- Urysohn's lemma
- Yoneda's embedding
- Reflection theorem

# The Mizar language

- The Fitch-Jaśkowski system (composite logic)
- Semantic correlates
- Types in Mizar
- Adjectives
- Structures and inheritance

# The structure of the MML

- Axiomatic basis
- Relationships between articles
  - ★ The concept of the environment
  - ★ References



# Writing Mizar articles

- Accommodator
- Verifier
- Step-wise refinement
- Formal proof sketches
- Filling gaps

# Maintaining MML

- Revisions
- Enhancers (relprem, relinfer, trivdemo, reliters, inacc, chklab, . . . )

# Arithmetic

- Strong arithmetic of real numbers had been introduced axiomatically

- The axiom:

$$k = \{i : i < k\}$$

has been added - with it one can prove that natural numbers are just finite ordinals

- The set of real numbers has been constructed
- Real numbers have been embedded into complex numbers

# Conclusions and future plans

- MML will be systematically revised using statistical data
- We need bigger and more diversified MML
- We want more mathematicians involved
- So, we need bigger, maybe specialized MML

We want to concentrate our efforts on a carefully chosen field of mathematics. Probably General Topology.