OpenMath Meeting 2–3 March 2002
Nice, France
(This was believed to be the 15th)

Present: John Abbott (JAA), Anna Bigatti (AB), Stephen Buswell (SB), Antonio Capani (AC), Olga Caprotti (OC), David Carlisle (DPC), Arjeh Cohen (AMC), Stéphane Dalmas (SD), James Davenport (JHD), Stan Devitt (JSD), Mike Dewar (MCD), Jean-Marc Fédou (J-MF), Marc Gaëtano (MG), Corrado Giromino (CG), Michael Kohlhase (MK), Steve Linton (SAL), Ursula Martin (UHM), Winfried Neun (WN), Peter Sander (PS), Mika Seppälä (MS), Andreas Strotmann (AS), Jouko Väänänen (JV) Stephan Watt (SMW), Victoria Wicks (VW).
Minutes by James Davenport

April 15, 2002

1 OpenMath day 2.3.2002 — morning

1.1 W3C Plenary session the previous week.

DPC spoke to this issue. He and SB observed the XML-Core working group on the Monday and Tuesday. This is looking at the future of XML. One issue is that of "entities for mathematical characters". For most of Thursday and Friday he attended the MathML group, but also XSL on Friday Morning. XSLT2 was being discussed, but there seem to be no implications for mathematics. The main thrust of the plenary day (Wednesday) was meant to be "web services", but the day was all about process rather than content. Several new working groups have been formed, mostly with blank sheets. The XML Protocol Activity is looking at SOAP, with a view to bringing out a SOAP2. The Web Services Description Activity is looking at WSDL, and in practice will probably adopt this.

As regards the MathML working group, they are explcitly *not* planning a MathML 3. 6 of the current audience were at the MathML meeting. This is a somewhat different charter from most W3C Working Groups. Since most browsers do not have native support for MathML, one needs to insert browser-specific strings into the document. The group will be making public a style-sheet that knows how to do this. MK gave a presentation on OMDoc. The possibility of producing a Schema was discussed. There was discussion

of Web Services — SB said that the main issue was the description level: the current proposals had 'A' knowing all about the semantics of 'B'. MCD pointed out that Mathematica thought that MathML was presentation, but Maple thought that MathML was content.

1.2 Interactions between OpenMath and MathML

SMW spoke to this issue, under the title of "Semantics-Preserving Transformations for Mathematical Markup". He listed the limitations of MathML: scope limited to K–12, limited precision of the semantics, Meta-knowledge, reflection and description of services. He pointed out that it possible to refer to OpenMath symbols in MathML-2 via the `csymbol` construct.

Since several browsers can display Presentation MathML but not Content MathML, it is important to be able to convert MathML-C to MathML-P, wthout losing the semantics of the MathML-C. Conversions to/from TEX would clearly be helpful, but the semantics implicit in the macro structure should probably be preserved.

He gave an example of algebra in a Boolean Ring, with MathML-C of

```
<apply>
   <and/>
   <apply> <xor/> a b </apply>
   <apply> <xor/> c d </apply>
</apply>
```

to be rendered via MathML-P as $(a + b)(c + d)$, but still be semantically recognised as a Boolean Ring. This could be done by wrapping the whole in a semantics tag, or by placing sematics tags on each operator or sub-tree. One can avoid the quadratic (MK: exponential) growth by means of adding tags, and giving the semantics to the tags.

He then gave a TEX example.

```
\newcommand\J[2]{J_{\1}(\2)}
$$
\J3z=\left(\frac8{x^2}-1\right)\J1z-4\J0z/z.
$$
```

Expanding `\J` would lose semantics. At UWO, they associate a mapping file with a TEX style file, so that `\J` and its arguments becomes an `<apply>` construct. Some of this work is on `http://www.orcca.on.ca/MathML`.

1.3 OpenMath Content Dictionaries

JHD spoke to this item. In particular, the polynomial CDs had been reviewed by JAA and Hans Schonemann. Numerous questions were raised, notably

- the change of name from `groebner_basis` to `groebnered` (which was made for consistency with `factored`) — `is_groebner_basis` was proposed, but JHD pointed out that this would imply a testing predicate;
- the rôle of `logic3` and its relations with ECC and OMDoc;
- the encoding of `ODEsolution` as a binder;
- the SI-completeness of the `units_imperial1` CD;
- the absence of a CD describing the U.S. units system — JHD thought that this was a task for NAOMI.
- JHD had mentioned an algorithms CD — JAA pointed out that there were different models of computation, e.g. quantum computing. JHD felt that this would be a different CD.

It was agreed to take these up directly with JHD in the informal sessions.

1.4 A Geometry CD

AMC spoke to this issue, a `plangeo` CD. He felt that symbols needed included `point`, `line`, `incident` and `configuration`. A point called A would be encoded as follows:

```
<OMA>
  <OMS name="point"/>
  <OMV name="A"/>
</OMA>
```

There is a symbol `type` for determining the type (line/point) of a geometric object. There is also a `exists` operation (returning a Boolean). The geometry package Cinderella can output its configurations (modulo the fact that circles are not yet defined) in OpenMath according to this CD.

AMC also demonstrated an "OpenMath shell", with conversions, say, to/from OpenMath and Mathematica.

1.5 Discussion 1

CDs MK called into question the current process "where JHD writes all CDs". JHD pointed out that his job description was "CD Editor" not "CD Author". DPC said that CDs should be available widely in an unrefereed state. OC asked why MK had not submitted his CDs. MK said that there was no open procedure for this, but MCD said that there was one on the OpenMath Society Web Site. JSD pointed out that, as an application developer, he would ask if there was a suitable CD, and if not, he would develop his own. There is a rôle

for the OpenMath CD is making even such an unofficial CD available. He summarised by saying that there was a kick-starting rôle and a monitoring rôle.

This developed into a general discussion on the availabilty of CDs and the structure of the Web site, and how we encouraged the use of OpenMath. In particular MG thought that we were should of implementations. SB pointed out that the Network did not have the resources to develop implementations. He felt that the companies would only ever implement MathML, because that's where the big bucks are. OC pointed out that, at Dagstuhl, the potential users there were surprised by the availability of the tools and libraries, and called for better publicity for these tools. OMC said that, although Mathematica did not support OpenMath, they were still able to use it fully via their API. SMW pointed out that Maple could export the MathML expression tree, and the API let you write a C programme that could traverse any Maple expression.

AMC was worried about the word `groebnered` — do we need a new word for every new concept. JAA pointed out that this name change was based on `factored`, so AMC shifted to `factored`. Why don't we just say that this polynomial–power construct multiplied out to the original. JHD pointed out that more was implied, i.e. that the polynomials were irreducible, and similarly for Gröbner bases. He said that maybe he should write FMPs for these constructors.

2 OpenMath day 2.3.2002 — afternoon

2.1 The MONET project

MCD spoke to this. This was to apply the "Semantic Web" concept to mathematical software and services. This will need:

- Mathematical Query Language;
- Mathematical Service Description Language;
- Explanation/Debugging support;
- Prototype infrastructure;
- Prototype applications and services.

The project will be a two-year investigative project, with a start date of 1 April 2002 (or possibly May). This project will liaise with the OpenMath Network via joint workshops. The consortium is NAG (sub-contractors Bath

and U.W.O.), Stilo (sub-contractor University of Manchester), Eindhoven (to include OC), and UNSA/INRIA.

The fundamental concept is that an OpenMath CD is a mini-ontology for the fragment of mathematics that is describes. This may need to be interfaces to general ontology mechanisms such as DAML+OIL. UDDI (University Description, Discovery and Integration) is very centralised, whereas systems like Napster and Gnutella are much less so. There are issues to do with user profiles and modelling, non-mathematical requirements such as privacy, subscription, payment etc.

2.2 OpenMath through the use of CoCoA

JAA spoke to this. CoCoA is a specialisd system, specialising in polynomials, and whose cornerstone is an efficient implementation of Buchberger's algorithm (an implementation of `<OMS name="groebner" cd="polyd"/>`). A new version is under development — should OpenMath be used for front-end/back-end communications. He ponted out that his data were very highly structured, unlike general expression trees, and hence the `polyd` CD was appropriate. `DMPL`, a list of dense multivariate polynomials, is often the fundamental type, but the ordering is attached to the individual polynomials, rather than the (common) ring specification. It could also be argued that a generic "homogenous list" constructor would be better than a specialised `DMPL`. He therefore suggested:

- A common layout syntactic_type(semantic_type,value_part);
- A "Homogeneous collection" operator HC(n,format,value$_1$,. . .,value$_n$) where the "format" would have placeholder objects (written `-`), so that `DMP(poly_ring_d(GF(5),2),...)` (with two terms)
  would be represented as
  `HC(2,term(OMATTR(-,type=GF(5)),-,-),1,5,4,3,2,1)`,
  representing the mathematical object $1 \cdot x^5y^4 + 3 \cdot x^2y^1$.
- This could lead to a distinction between "fat" (typed) objects and "thin" (untyped, because the enclosing structure implied the type) objects.

He concluded by saying that OpenMath needed better worked examples, as well as a clearer distinction between general trees and highly structured values. His concrete suggestions provoked a lively debate, partly on the more general question of how different CDs describing the same objects could (or should) be repeated. SMW said that this recalled the presentation/content debate in MathML. AS said that the MP protocol had a similar syntax/semantics debate.

2.3 Discussion 2

JHD introduced the results of a lunch-time discussion between himself, DPC, SB and MK. This had concluded that, given the existence of `csymbol` in MathML-C, and the fact that this can be given bound variables, it is possible to encode OpenMath in MathML-C. He noted that part of the MathML working group's new charter was inter-operability, and one question was what the "official" nature of such an encoding should be. DPC said that, in the previous OpenMath project, there was a compatibility document written by JHD, and he had written the corresponding XSLT style sheet.

AMC whether this helped with OpenMath rendering. SMW though that it did, since there were MathML-C → MathML-P renderers. DPC disagreed, since the default MathML-C rendering for `csymbol` was prefix form.

MK argued for throwing away the current standard XML encoding in favour of this. He thought that this would buy more tool support. There is a considerable publicity machine behind MathML. This would let us concentrate on semantics. AS said that, in this scenario, the translation was not perfect, but the lunch-time group convinced him otherwise. There was the further question of whether `<OMS cd="arith1" name="plus"/>` should map to `<plus/>` or `<csymbol definitionURL="http://www.openmath.org/OpenMath/arith1"/>`. JHD pointed out that, if we did map to `<plus/>`, then we could regard MathML-C as "OpenMath-LITE". SB pointed out, that, if we did this, `defint` would disappear as a separate symbol. MS supported MK's proposal. DPC argued that the current XML encoding was a natural linearisation of the abstract tree, and the MathML-C encoding would not be. AMC argued against this replacement, saying that it was not a good time to disturb current users.

JHD noted that such a change to the standard would be a matter for the OpenMath Society, and he proposed that we should explore the technical issues for conversion XSLT in both directions, and then get the MathML group to adopt this as an interoperability statement, which would get us a certain amount of W3C blessing. SB said that this might be possible, and would allow us to state "OpenMath renders natively in IE6 and Mozilla" (though the statement would be intellectually trivial).

2.4 A Content Language for Distributed Reasoning Systems

CG[1] spoke to this. This was work developed when he was at Genoa. He first introduced a Logic Broker Architecture. Servers would register with the Logic Broker, and clients would ask the Logic Broker to find servers that could answer their queries. The LBA needs a Logic Service Matcher.

The design requirements were:

- interoperability for development, communications and logic;

[1]Now at Saarbrücken.

- extensibility;
- accessibility.

This led them to use a CORBA layer with a Logic Broker lying underneath it. OpenMath then becomes the common content language. There is a major problem with findng a common logic between different clients and servers. They are using an IDL description (rather than the XML encoding) and a CORBA implementation of OpenMath objects.

They used CoCoA-3 and a couple of theorem-provers. Logic matching was a problem — CoCoA would advertise a `factorise` function, and the client would ask for this, so the Logic Service Matcher was textual.

SB asked whether CORBA could be replaced by SOAP. CG said that you would also need to use a layer such as RPCJava, which would cause problems with the interoperability of communications. AS pointed out that `<OMI>` was represented by a `long` value, rather than arbitrary precision.

2.5 Dependable Mathematics and Computational Logic

UHM spoke to this topic. This talk was more about providing mathematical services, rather than interfacing them. Having described computational logic, she noted that the impact of this on mathematics was slight, since proof $\neq$ mathematics. The main goals of this field are really in automated verification, e.g. chip design, air traffic control, protocol verification, avionics code certification. Her goal was to identify and build useful generic computational logic services that could enhance assurance or extend the power of familiar paradigms in numerical (and to a lesser extent symbolic) computation.

Maple 6 is untyped, ignores side conditions, has semantics based on differential algebra. PVS supports hightly automated theorem proving or simple proofs. UHM's team *et al.* have a large base of tactics and lemmas. This enhanced PVS has semantics which are standard continuous mathematics. The interface technology was Tcl/Tk using the Maple/C link. After Maple has produced a result, Maple/PVS can check that the solution exists under certain side conditions. Equally, for verified integral table look-up, can use PVS to discharge the side-conditions in a properly guarded integral table. Work had also been done at St. Andrews to generate verification side-conditions for Axiom/Aldor.

Current work focuses on control engineering applications. They are working to identify the computational logic tools and knowledge bases required, e.g. complex numbers, special functions, differential-algebraic equations etc. AS pointed out the example of $\left(\int \frac{1}{1+x^2}\,\mathrm{d}x\right)'$, which Maple cannot prove equal to $\frac{1}{1+x^2}$.

There was a debate on PVS versus Maple in the context of integration. AMC suggested that Maple should be enhanced, but UHM pointed out that

it still would not provide the level of guarantee that PVS did. SMW asked whether she would not end up re-implementing Maple in PVS. Maple had debated keeping an audit trail of side-conditions, but these tended to be exponential in size.

2.6 Discussion 3

MK opened a discussion on the future of the OpenMath Standard. In particular, is it set in stone now? MCD pointed out that the Thematic network is committed to delivering annual revisions, which could just be errata. At Linz, it was decided that fundamental changes were not required.

3 OpenMath day 3.3.2002 — morning

3.1 Helsinki Learning System

MS spoke to this. MS explained that they were not currently using OpenMath, but wished to do so in the future. MS reminded the meeting of the hype around on-line learning a few years ago. The events predicted have not happened, a new paradigm has not properly developed, and broadband access at home is still not wide-spread (3 out of 24 in the room).

At Florida State University, they have used databases of elementary problems to allow students to perform self-assessment in low-level mathematics courses. This has reduced the failure rate by 50%. At Helsinki, they wish to follow a similar path. There are several textbooks in use, but the content is similar, so that, via a Course Content Dictionary, we can choose the set of problems relevant to a given section of a given book. The system is multi-lingual, and the exercises are problem trees with branches to "correct", "incorrect" and "don't know". MK pointed out the IMS standard, and asked MS why he wasn't using it. MS said that the project started before IMS became wide-spread, and it did not support all the features they wanted. There is an authoring tool. The mathematics is currently in TEX, but they are contemplating conversion to MathML or OpenMath. The current application is (uni-variate) calculus, with Maple 7 being used as a supporting module. The original idea was to replace textbooks, but it now seems more realistic to complement them. Further collaboration are planned with UNED (Spain) and a Mexican group.

He said that he originally believed that "bringing mathematics alive" was the key, but had come to the conclusion that the database of problems was the key. DPC believed that, in this sort of context, translating the TEX into, say, MathML, would be relatively easy.

3.2 A Language for Brokering Mathematical Services

OC spoke to this item. She pointed out that MCD had already covered much of the background of this topic. This was a two-year project (since Dec 2001) funded by the Austrian Science Foundation. The project's first focus will be on service description languages. Currently, mathematics on the Web is nearly all intended for human operation, rather than autonated services: barely machine-readable, and certainly not machine-understandable (no metadata etc.). There was a debate on "service description", and how humans found services — often by recommendation in practice.

They are currently using Axis: `http://xml.apache.org/axis/`. They also wish to model mathematical services in WSDL. They are wondering whether to define a Mathematical Services Description Language from which to gnerate WSDL and the appropriate OM descriptions.

3.3 A Categorial Type Theory for OpenMath

AS spoke to this item. He said that several different type systems exist for OpenMath. There are different meanings for `OMA` and `OMBIND`. His proposal was sislightly more complicated than STS, but less so than ECC. The λ-term $(\lambda x.\lambda y.fxy)ab$ would be represented categorically as $((X\backslash(Y\backslash((F \cdot X) \cdot Y) \cdot A) \cdot B)$ with $\backslash$ representing application and $\cdot$ representing application. In his system, `OMA` works as in STS, as application. The type of sin becomes $\mathbf{C}/\mathbf{C}$, and plus has type $\mathbf{C}/_\omega\mathbf{C}$ where $/_\omega$ is syntactic sugar for "n-ary" (he does not distinguish between "n-ary" and "n-assoc". The type of $\texttt{OMBIND}(a, v_1, \ldots, v_n, e$ is $A \cdot (E/V_n/\cdots/V_1)$. He noted that OpenMath defined[2] a currying rule for binders. He could also assign types to attributes.

We might extend the scheme by defining built-in type hierarchies. A recent development of categorial logic, composition, would let one type, say, $(\sin^2 + \cos^2)$ as $\mathbf{C} \to \mathbf{C}$. He identified the following problems.

- The currying of `OMBIND` operators. There are problems of scope capture.
- Conversely, `OMA` are not curried.
- Signatures are not curried. But $f(a, b) = (\lambda x.\lambda y.f(x, y))ab$, and the right-hand side can have its binder curried.

He proposed that binders are operators, and vice versa. He proposed the following.

- Any OpenMath object that takes a single function as its argument can be used as a binder.

[2]This was discussed at the previous meeting (Linz), where several members wanted to abolish this rule.

- Any binder can be used as an OpenMath operator.

He furthermore claimed that $\forall x \in \mathbf{R}$ was ambiguous: was it typing x or qualifying the values that x might take, e.g. $\forall x \in (\mathbf{R} \setminus \{0\})$.

4 Embedding OpenMath into Documents: lessons and suggestions from the OMDoc Experience

MK spoke to this issue. OMDoc has three levels of modeling.

1. Formula-level: OpenMath or MathML-C.
2. Statement-level: e.g. recursive definition of "+". Also theorems, proofs, examples etc. The text elements are `context` and `CMP`. There are signatures of `symbol` and `definition`. A proof is a sequence of FMPs, bound by `Hypothesis`, `Deduction` etc. There can be formal and informal statements, bound by `xref=` constructs. This supports multi-linguality. Note that he has introduced new attributes `id=` and `xref=` to every OpenMath object (except ground elements). The current implementation is not based on XLINK (though it should be), and the MathML uses of these attributes are subtly different.
3. Theory-level: theories can be inherited via symbol-mapping.

He called for primitive support in OpenMath for records, i.e. generalised tuples. In particular, he proposed taking `OMATP` and making it first-class and n-ary (under the name of `OMSTRUCTURE`). In response to a question, he gave a concrete example of a monoid as a set with a binary operator and an identity. DPC asked why this couldn't be done via a CD. MK said that it could be done, but most languages had chosen to make it a linguistic feature.

OMDoc has a means of specifying default presentation, such as "infix with precedence 200". This is encoded in XSL. He felt that OpenMath hadn't really addressed the issue "how do I find this content dictionary"? Suppose there are multiple repositories? MCD asked for a concrete example. MK is currently using RDFDictionary and RDDL.

He also thought that CDs could better be written in OMDoc. Then ActiveMath could be used as a front-end.

5 OpenMath Society Annual General Meeting 2002

AMC opened the meeting at 12.15.

1. Antonio Capani was elected a member (3 workshop rule), and Paul libbrecht (6 months work rule). 14 members were present. AMC was

elected President of the meeting. JHD was elected minute-taker. MK was nominated scrutineerer, and DPC as minute-checker.

2. A report was received for the Fiscal Year 2001. There had been expenses for the `openmath.org` domain (encumbered in the previous year). It was pointed out that the Society would soon be in deficit. AMC stated that we should raise about 200-300 Euro/year, at the current rate of expenditure (which was likely). AS reminded the meeting that the constitution allowed for corporate membership.

3. The issue was membership fees. AMC floated two routes:

   (a) an annual membership fee;

   (b) a participation fee at the OpenMath Society meeting.

   SMW proposed, and SB seconded, the participation fee. This was approved.

4. Membership of the Executive Committee. The absent members were Gaston Gonnet and Stephen Braham. The EC will contact GHG, the father of OpenMath, and ask him to continue. SB has not been active for some while. The EC proposed thanking him for his past activity, and proposed MK as a replacement. This was seconded by SB, and accepted.

5. Election of Auditors (currently AS and TH). AS was asked to continue, and AC was asked to become an auditor. This was approved.

6. OpenMath Logo. One had been constructed by AC. SMW said that the capitalisation "OpenMath" should be preserved. It was proposed to make the other letters slightly smaller than the 'O' and 'M'. As modified, the logo was approved to acclamation.

7. Any other Business.

   (a) AMC: An improved Web site has been promised for a long time, and it is hoped to be ready by Easter. He suggested that, while waiting for this, we should write various promotional Web pages, e.g. a history of OpenMath, an OpenMath manifesto (including relationship with MathML), a page of relevant links, and so on.

   (b) The just-completed workshop had noted the possibility of conversion between OpenMath and MathML-C, and the meeting noted this and hoped that a more detailed proposal would emerge. It was noted that calling it an "encoding" raised various subtle political questions.

(c) AC raised the question of CD development. AMC said that proposed CDs should be mailed to the CD Editor (JHD). AMC and AS said that the validation tools had not worked for them. MK said that there is a perception that the OpenMath Society is the sole author of CDs, and this had to be changed.

(d) There were various proposals for changes to the standard: JAA, MK(4) and AS. The question of the procedure for such changes was raised. MCD said that there were several documents: standard, type systems, guidelines etc. JHD proposed that a formal rationale and detailed proposal for change should be circulated a month in advance, and the editor(s) of the relevant document(s) should circulate comments at least two weeks in advance. This was approved.

(e) Date of next meeting. MCD noted that there would be an MKM meeting in February 2003, which would make a natural OpenMath meeting. The natural choice for the next Thematic Network meeting would be Calculemus (3–5 July) or ISSAC (10–12 July). The MathML Meeting in Chicago in June 2002 was mentioned — there were some OpenMath papers there, but probably not enough for an OpenMath session as such. SMW (Programme Chair) said that there could be an OpenMath panel. It was noted that AMC and MK would collaborate on an OpenMath tutorial.

The meeting closed at 13.10