Q1 Solve the differential equation.

$$(x^2-1)\frac{dy}{dx} = -(x^2+1)y$$

for $x<1$

$$(x^2-1)\frac{dy}{dx} = -(x^2+1)y$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{x^2+1}{x^2-1}$$

$$= -\frac{x^2-1+2}{x^2-1}$$

$$= -1-\frac{2}{x^2-1}$$

Fine to here

$$\frac{d}{dx}\left(\frac{1}{x}\ln|x^2-1|\right) = \frac{1}{x}\left(\frac{2x}{x^2-1}\right) - \frac{1}{x^2}\ln|x^2-1|$$

$$\Rightarrow \int\frac{1}{y}\,dy = \int\left(-1-\frac{2}{x^2-1}\right)dx$$

$$\Rightarrow \ln|y| = -x - \frac{1}{x}\ln|x^2-1| + c$$

You should have used partial fractions, or at worst a hyperbolic trig. substitution!

$$\Rightarrow e^{\ln|y|} = y = e^{-x-\frac{1}{x}\ln(x^2-1)+c}$$
$$= e^c e^{\ln(x^2-1)^{-\frac{1}{x}}} e^{-x}$$

$(x^2-1>0$ for all $x>1)$

So,

$$y = A(x^2-1)^{-\frac{1}{x}}e^{-x} \quad \text{where } A=e^c$$

Find the general solution to the differential equation

$$(x^2-1)\frac{dy}{dx} = -(x^2+1)y,$$

given x>1.

Your last answer was interpreted as:

$$A\cdot(x^2-1)^{-[1/x]}\cdot e^{-x}$$

Incorrect answer.

Your solution is incorrect. Substituting your solution into the left-hand side of the differential equation gives

$$(x^2-1)\frac{dy}{dx}=(x^2-1)\cdot\left(\frac{e^{-x}\cdot\left(\frac{\ln(x^2-1)}{x^2}-\frac{2}{x^2-1}\right)\cdot A}{(x^2-1)^{[1/x]}}-\frac{e^{-x}\cdot A}{(x^2-1)^{[1/x]}}\right).$$

Whereas substituting your solution into the right-hand side of the differential equation gives

$$-(x^2)+1)^*y=\frac{(-(x^2)-1)\cdot e^{-x}\cdot A}{(x^2-1)^{[1/x]}}.$$

Answer: A*(x^2-1)^-(1/x) * e^-x

	Working (entry)	Working (verified)	Comments
1		For x>1, solve the differential equation	
2		$\left(x^2-1\right)\frac{dy}{dx}=-(x^2+1)y$	
			Dividing by (x^2-1) Dividing by y
3	`1/y diff(y,x)=-(x^2+1)/(x^2-1)`	$\Rightarrow \frac{1}{y}\frac{dy}{dx}=-\frac{x^2+1}{x^2-1}$	
4	...	$\Rightarrow \frac{1}{y}\frac{dy}{dx}=-\frac{x^2-1+2}{x^2-1}$	
5	...	$\Rightarrow \frac{1}{y}\frac{dy}{dx}=-\frac{x^2-1}{x^2-1}-\frac{2}{x^2-1}$	Integrating with respect to x
6	...	$\Rightarrow \int\frac{1}{y}\frac{dy}{dx}dx=\int\left(-1-\frac{2}{x^2-1}\right)dx$	
7	...	$\Rightarrow \int\frac{1}{y}dy=\int\left(-1-\frac{2}{x^2-1}\right)dx$	Evaluating
8	...	$\Rightarrow \ln\lvert y\rvert=-x-\frac{1}{x}\ln\lvert x^2-1\rvert+c$	Taking exponentials
9	...	$\Rightarrow e^{\ln\lvert y\rvert}=e^{-x-\frac{1}{x}\ln\lvert x^2-1\rvert+c}$	Manipulating indices
10	...	$\Rightarrow y=e^c e^{-\frac{1}{x}\ln\lvert x^2-1\rvert}e^{-x}$	Manipulating logarithms
11	...	$\Rightarrow y=e^c e^{\ln\lvert x^2-1\rvert^{-\frac{1}{x}}}e^{-x}$	
12	...	$\Rightarrow y=e^c\left(x^2-1\right)^{-\frac{1}{x}}e^{-x}$	Substituting $A=e^c$
13	...	$\Rightarrow y=A\left(x^2-1\right)^{-\frac{1}{x}}e^{-x}$	**Solution**

Incorrect answer; error in integration of step 7.
Subsequent working correct up to error.