

*Content Markup:  
Principles and Consequences*

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# *Introduction*

Presentation of results from my dissertation  
*“Content Markup Language  
Design Principles”*

- Ladislav J. Kohout, major professor
- Mika Seppälä, R. van Engelen, K. Gallivan, H. Levitz, committee members
- official date May 2003, FSU CS
  - [www.cs.fsu.edu/research/reports](http://www.cs.fsu.edu/research/reports)
  - [etds.lib.fsu.edu/](http://etds.lib.fsu.edu/)

## *Introduction (ctd.)*

- ❖ Focus on ideas with results that may influence development of OpenMath 2.0
- ❖ including those that have influenced development of MathML 2.0, 2nd ed.
- ❖ To be continued this afternoon with concrete proposals for OpenMath 2.0

# *Research Topic*

- ❖ Understanding Content Markup Language Design
  - *well...* “towards a better understanding of...”
  - because existing language designs have been flawed
  - ... due to lack of a deeper understanding
- ❖ Approach Based on an Observation
  - Content markup languages are knowledge communication languages for heterogeneous systems
  - there is only one known high-quality solution to the knowledge communication problem: human language

# *Research Ansatz*

## ❖ => The Linguistics Approach

- Linguists (and others) have been studying “engineering solutions” of human language for a long time, with impressive results
- Proposal: transfer “engineering solutions” to content markup language designs

## ❖ But: How do we “prove” this works???

- Formal proof clearly impossible...
- *The proof of the pudding is in the eating*

# *Research Method*

- ❖ Application of select tools from linguistics to content markup language design
  - Language architecture: layers & components
  - Compositionality Principle
  - Categorical Semantics
- ❖ Successful transfer of these non-trivial “corollaries” supports main “conjecture”
  - + Outlook to as yet untried tools adds weight

# *Table of Contents*

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- ❖ The Compositionality Principle
- ❖ Categorical Semantics
- ❖ Conclusions

# *Linguistics Parallel: Motivation*

- ❖ Human language universals
  - developed under intense evolutionary pressure
  - => provide a “good” engineering solution
    - => use “principles”, but ignore “parameters”
  - Human language solves similar problem to content markup
    - communication of meaning between similar but different intelligent agents
    - => study design principles of language in order to design good content markup languages

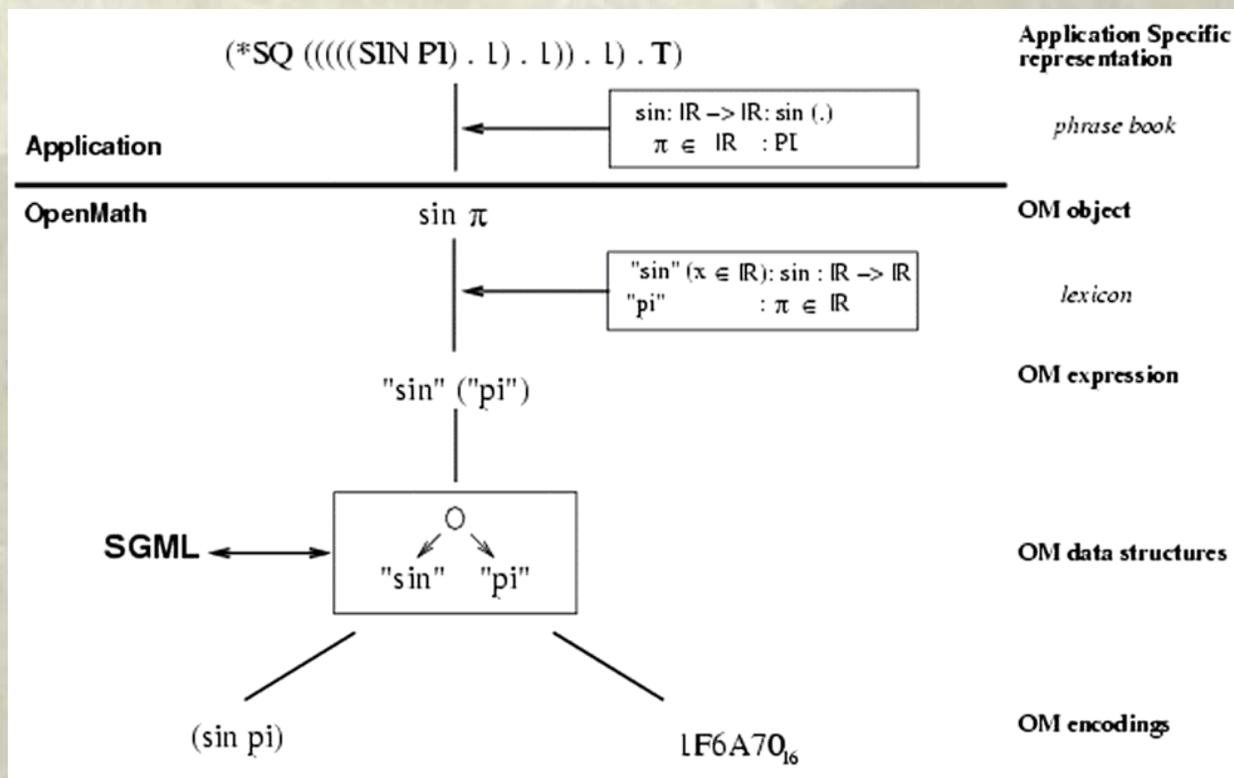
# *Linguistics Ansatz: Language Layers*

## ❖ Linguistics background

### – Language components:

- Morpho-syntax,
- Syntax,
- Structural (“categorical”) semantics,
- Lexical semantics,
- Pragmatics,
- Semiotics

# Application: Language Layers for OpenMath



- published in "OpenMath Objectives" ('95/98)

# *Linguistics Ansatz: Syntax Layer*

- ❖ Linguistic background (& proposals)
  - “X-bar” : “typed” tree structure  
( (head arguments...) modifiers... )
    - cf. OpenMath ((head arguments...) attributions...)
    - head determines “type”
    - modifiers ~ named arguments with defaults
  - “Government and Binding”
    - syntax for scopes (cf. OM Binder, MML <bvar>)
    - syntax for co/cross-references (cf. MML id=, ref=)

# *Linguistics Ansatz: Syntax- Semantics Interface*

- ❖ Linguistics Background
  - Compositionality Principle (*aka* Frege-Prinzip)
    - “Meaning of compound expression is function of syntactic composition rule and meanings of parts”
    - Research principle underlying Formal Semantics
    - Many applications in CS
  - Categorical Semantics
    - Categories of lexical items with identical behavior
    - Meaning category from category of parts & syntax
    - Categorical type logics/ type systems

# *Linguistics Ansatz: Pragmatics*

## ❖ Linguistic Background

– syntactic categories correspond roughly to language layers

– NPs - who, what... : static semantics

– VPs - doings: dynamic semantics (?)

– IPs - judgments: pragmatics?

• Words can systematically shift levels

• categories can all be nested inside each other

## ❖ Content markup currently ~ Noun Phrase

– KQML, OMDoc ~> VP (actions, actors)

• mutual inclusion property still missing

# *Compositionality*

## ❖ Compositionality

- in CS usually understood to constrain semantics given syntax
- in philosophy of language, “systematicity” only possible given compositional world view
- in linguistics, compositional semantics and syntax constrain each other
- here: allowable syntactic structure constrained by intended semantic structure

# *Compositionality and Language Design*

- ❖ “Meaning of compound is function of meaning of parts and syntactic construct”
  - often: “exists homomorphism from syntactic algebra to semantic algebra”
  - hence: distinct semantic constructors require distinct syntactic constructors
  - usual ingredients: numbers, variables, names; application
  - also needed: variable binding, typing syntax

# *Compositionality and language analysis*

- ❖ Compositionality analysis of an example
  - determine semantic decomposition(s)
  - determine distinct semantic constructors
    - many, but not limitless possibilities
- ❖ Analysis of existing languages
  - determine if systematic representations of semantic constructors exist
- ❖ Design of new language
  - construct homomorphism

# *Compositionality and Variable Binding Syntax*

- ❖ Variable binding has special semantics
  - cannot be reduced to combination of other regular language ingredients and application
  - $\Rightarrow$  need special syntax
- ❖ Do knowledge communication languages have systematic special binder syntax?
  - Yes: OpenMath, MathML
  - No: KIF, CA user languages
    - But KIF 3 defines lambda as special syntax

# *Compositionality and Higher-Order Operators*

- ❖ Special syntax for variable scoping
  - $\Rightarrow$  “parts” must include body and bound vars
  - (more parts are possible, e.g. binder)
- ❖  $\Rightarrow$  Do specific language ingredients that represent variable binding require these as necessary parts?
  - Counter examples: KIF “setof”, MathML “min”
- ❖ When not: construct examples with errors

# *Compositionality: Practical Consequences*

- ❖ Found errors in KIF 3.0, dpANS KIF, MathML
- ❖ OpenMath Binding Objects explicitly added to improve compositionality (S. Watt)
- ❖ Languages with explicit typing require special type assignment syntax (missing in all content languages we have looked at)

# *Categorical Types*

- ❖ analysis tool for content markup languages
  - has been applied to mathematical formulas from 1935!
- ❖ ~ type-level generalization of  $\lambda$  calculus called Lambek calculus
  - application, abstraction, reduction rules...
  - types of atoms “ignored”/ factored out
  - unification of concrete types left as an SEP
  - interaction between categorial (structural) and concrete (lexical) type system generally benign
    - Dörre, Manandhar: On constraint-based Lambek calculi, 97

# *Categorical Types: Application to OpenMath*

- ❖ Categorical types for OpenMath
  - proposal of full categorical type system
    - compatible with existing systems
    - compositional categorical type assignment function for all OM Object constructors (application, binding, attribution, error)
  - flushed out and fixed severe OM spec error
    - current syntax of OpenMath cannot allow intended Currying semantics
    - problem traced to extra “part” of Binding object

# *Categorical Types:*

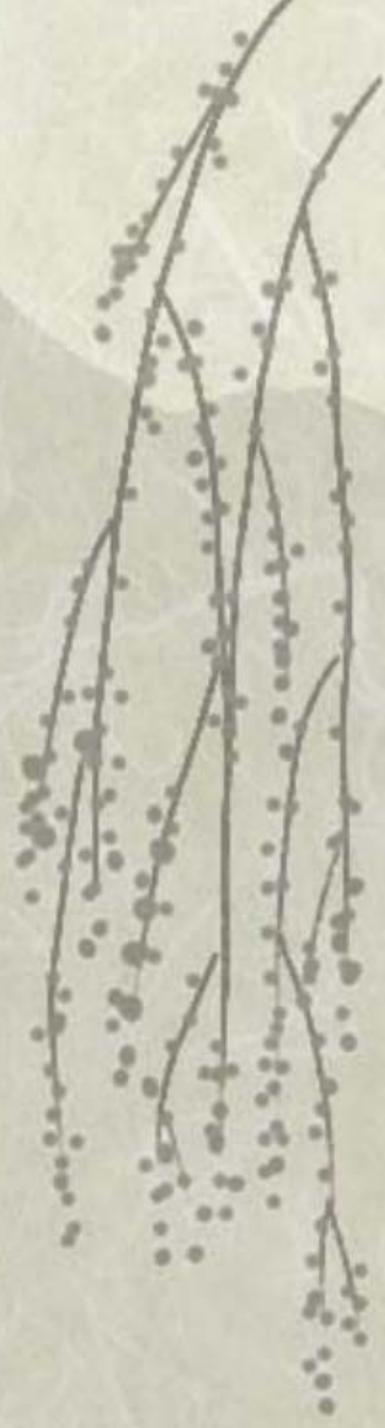
## *Application to MathML (to do)*

- ❖ MathML 2.0, 2nd ed, finally ready for categorical type system (10/2003)
  - Special syntax for variable binding, domain-of-application...
  - Applies systematically to any operator, not just a few
  - Systematic correspondence between „functional“ and „binder“ usage patterns conforms to our categorical type system view



# *Principles*

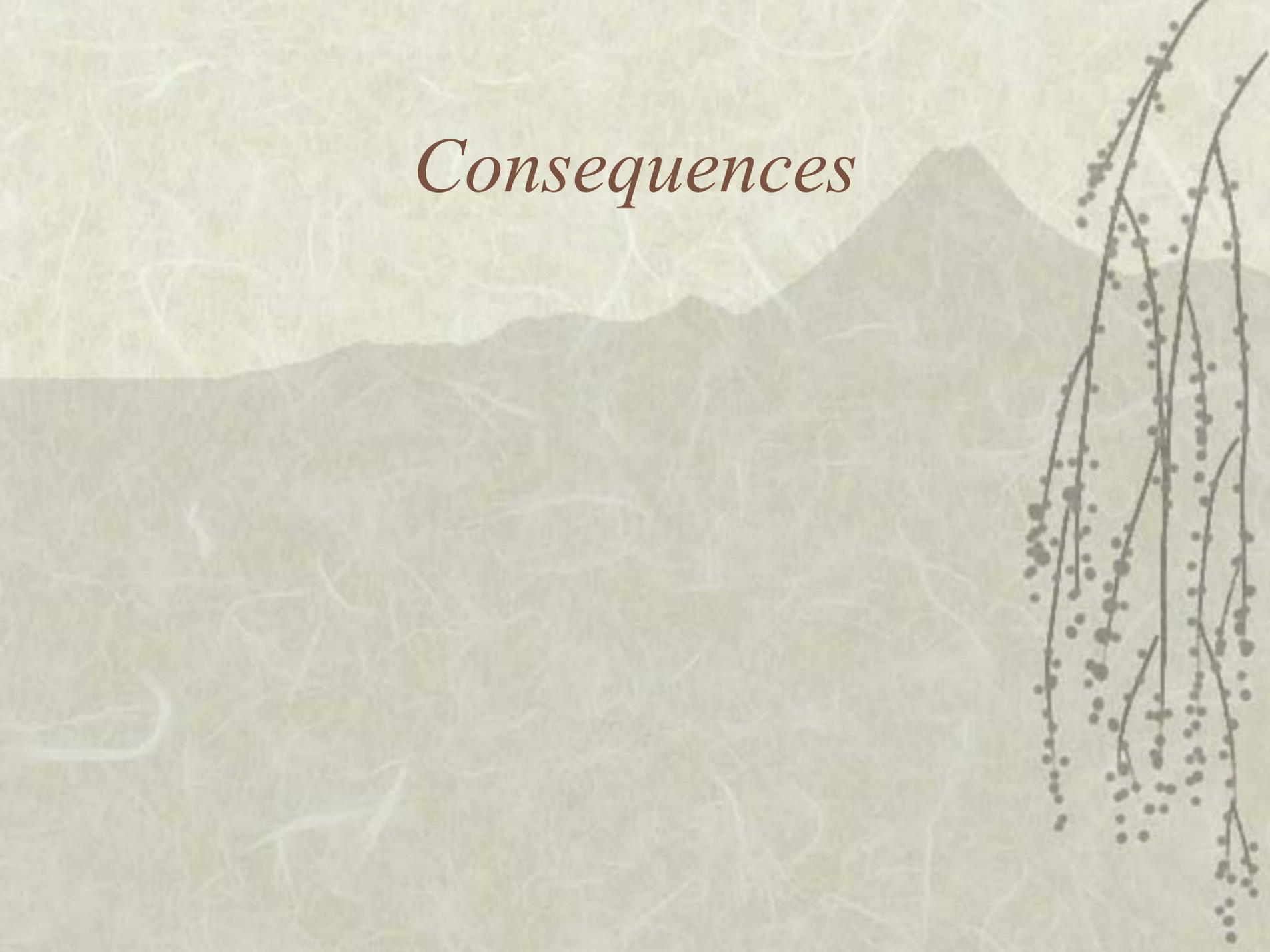
- ❖ Compositionality Principle
- ❖ Radical Lexicalism
- ❖ “Categorial” Semantics
  
- ❖ Linguistics Parallel



# *The Compositionality Principle*



# *Consequences*



# *Consequences ...*

## *from Compositionality Principle*

- ❖ Every class of qualitatively different semantic constructs requires its own special syntactic construct
  - atomic: variable, name, strings, numbers...
  - structural: application (positional and **named arguments**), binding, **typing** information
  - pragmatic: command, question...

# *Systematicity*

- ❖ If a class of concepts is open-ended, it should be handled systematically
  - MathML 1 --> 2: make binding and domains of application available beyond closed set of ops
  - MathML 2, 2nd ed.: make them available systematically, including equivalence of functional and binder formulation of generalized quantifiers
  - OM2 draft: binder symbols are regular ops, too!

# *Lexicalism*

- ❖ Clean factorization of semantics into
  - structural (a.k.a. “categorical”) semantics
  - and lexical semantics (“ontologies”)
- ❖ meaning(s) of a word is lexical entry
  - complex types/semantics as context specs
- ❖ semantic interpretation or type inference rules exclusively in structural terms
  - lexicon does not allow adding rules

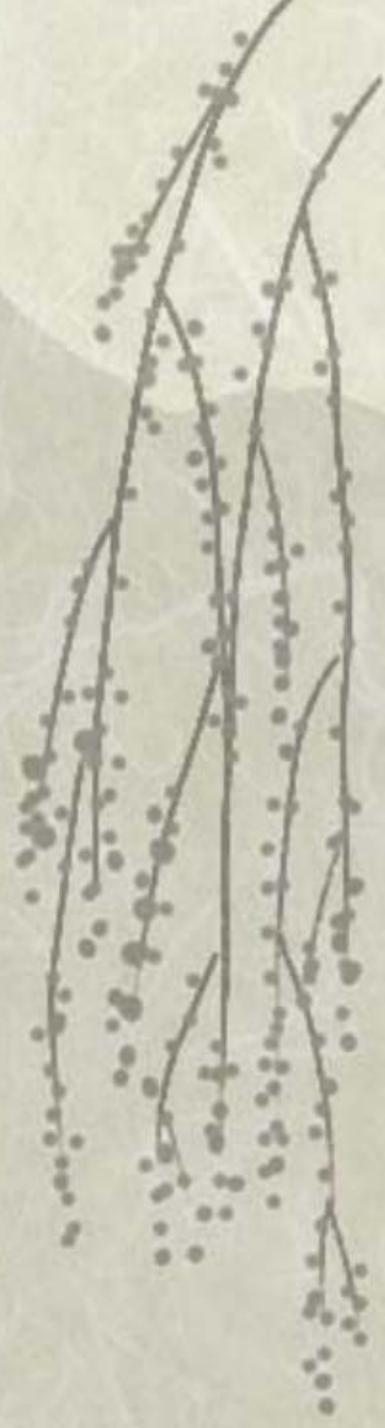
# *Lexicalism*

- ❖ Lexical type theory “orthogonal” to structural (“categorical”) type theory
  - Result from formal semantics (linguistics) lit: For a large class of categorical type theories (L2 and lower) and a large class of lexical type theories (lattice), their combination is very well behaved (e.g. decidability depends on lexical “plugin” type theory, *not* on categorical “framework” type theory)

# *Categorical Types*

- ❖ L2 encompasses
  - application and abstraction types
  - unification over type variables
  - currying (and much more)
  - no quantification over type variables (!!!)
  - no explicit typing (lexical typing only)
  - no(?) domain-of-application
- ❖ result applies to simpler theories, too

# *Proposals*



# *Cleanup*

- ❖ Make sure standard encodings can
  - encode all OM objects
    - remove arbitrary size limits in binary encoding
  - do round-trip encoding
- ❖ compatibility with MathML v 2 ed 2
  - equivalence of uses of binder symbols in application or binding objects
  - domain-of-application

# *Standardize Formal Structural Type System*

- ❖ Extend STS to become full-fledged standard “categorical” type system for OpenMath
  - we can (but don’t have to) define currying properly here!
  - Compatible with proposal for semantic attrs.
  - Equivalence of functional and binder uses
    - compatible with MathML 2, 2nd ed.
    - incompatible with some OM2 proposals!
- ❖ Possible, but not necessarily trivial

# *A Consequence for Types*

- ❖ Every type theory for OpenMath must extend the Standard Structural Type System
- ❖ Common fundamental type constructors
  - abstraction (mapping, n-ary mapping)
  - application (reduction)
  - natural numbers, reals, complex numbers...
  - type descriptor (e.g.)
  - CD or required entries in type theory def., e.g.

# *“Categoriality” property*

- ❖ Prove that extended STS works properly with embedded type systems
  - along the lines of existing proof of compatibility of L2 type logic with type lattices
  - potential problems:
    - explicit type assignments
    - n-ary operators

# *MathML compatibility*

- ❖ Common type theory may serve as basis for formal proof of compatibility between OpenMath 2.0 and MathML 2.0, 2nd ed.
  - Requires research!

# *OpenMath Layers*

- ❖ Consider re-introducing an extra OpenMath layer as originally proposed in “Objectives”
  - intermediate layer defined as structural (“categorical”) semantics layer

# *Reconsider Binding Objects*

- ❖ It turns out that the binder argument to a binding object complicates its semantics considerably
  - e.g. makes it impossible to define a currying rule without introducing a categorial theory
- ❖ Consider pros and cons of replacing binding objects by lambda objects.
  - These lend themselves naturally to currying